Пирамида

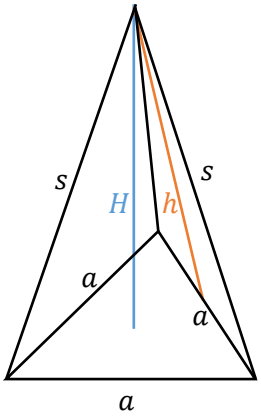
Права правилна тространа пирамида

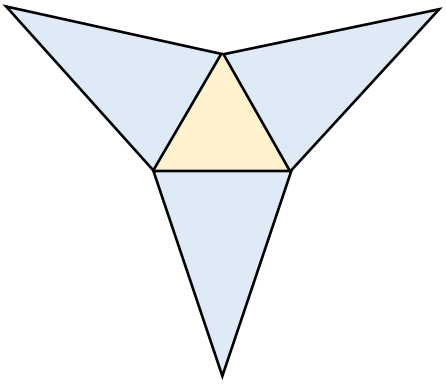
a – основна ивица,

s – бочна ивица,

H – висина пирамиде

h – висина бочне стране или апотема



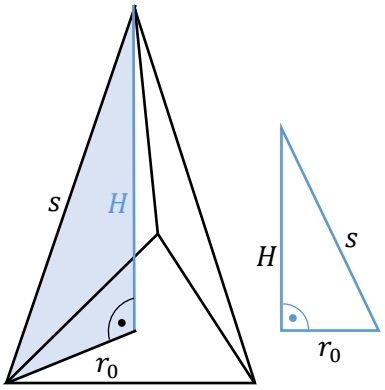


$$B = \frac{a^2\sqrt{3}}{4}$$

$$M = 3 \cdot \frac{ah}{2}$$

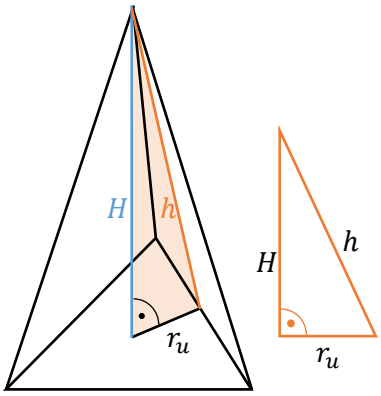
$$P = B + M$$

$$V = \frac{1}{3} \cdot BH$$



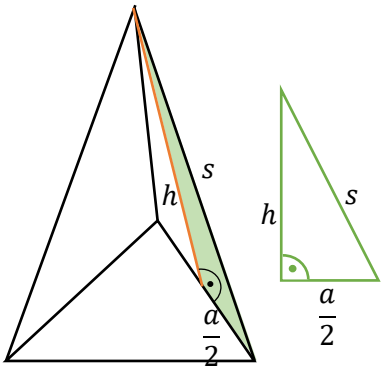
$$s^2 = H^2 + r_o^2$$

$$r_0 = \frac{a\sqrt{3}}{3}$$



$$h^2 = H^2 + r_u^2$$

$$r_u = \frac{a\sqrt{3}}{6}$$



$$s^2 = h^2 + \left(\frac{a}{2}\right)^2$$

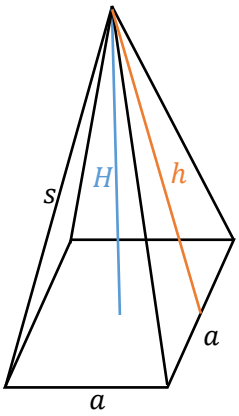
Права правилна четворострана пирамида

a – основна ивица,

s – бочна ивица,

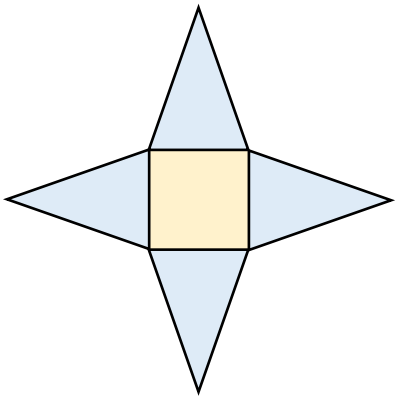
H – висина пирамиде,

h – висина бочне стране или апотема



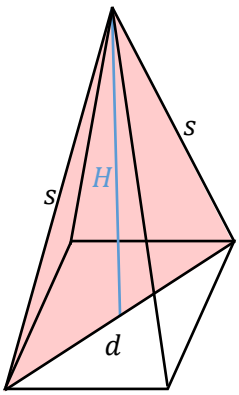
$$B = a^2$$

$$M = 4 \cdot \frac{ah}{2}$$



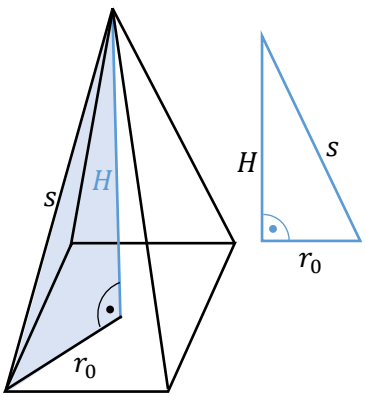
$$P = B + M$$

$$V = \frac{1}{3} \cdot BH$$



$$P_{dp} = \frac{dH}{2}$$

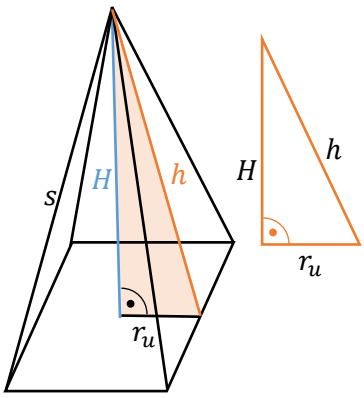
$$d = a\sqrt{2}$$



$$s^2 = H^2 + r_o^2$$

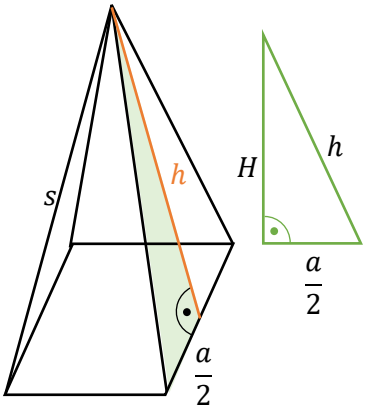
$$r_0 = \frac{d}{2}$$

$$d = a\sqrt{2}$$



$$h^2 = H^2 + r_u^2$$

$$r_u = \frac{a}{2}$$



$$s^2 = h^2 + \left(\frac{a}{2}\right)^2$$

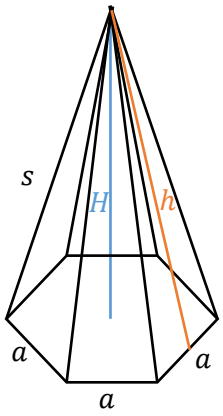
Права правилна шестострана пирамида

a – основна ивица,

s – бочна ивица,

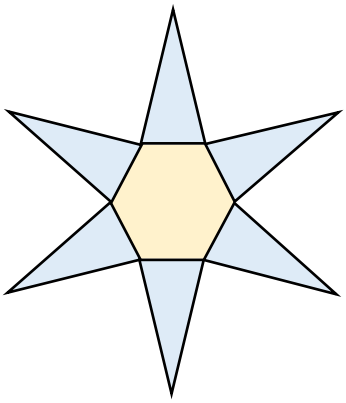
H – висина пирамиде,

h – висина бочне стране или апотема



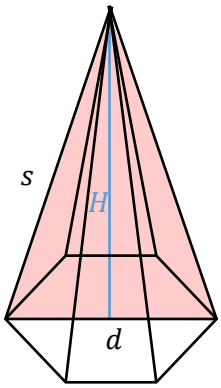
$$B = 6 \cdot \frac{a^2\sqrt{3}}{4}$$

$$M = 6 \cdot \frac{ah}{2}$$



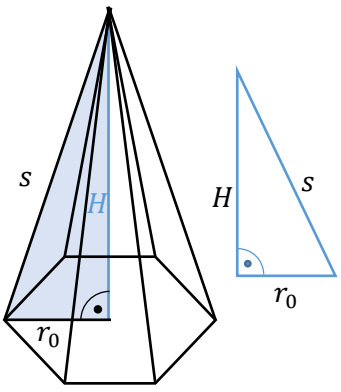
$$P = B + M$$

$$V = \frac{1}{3} \cdot BH$$



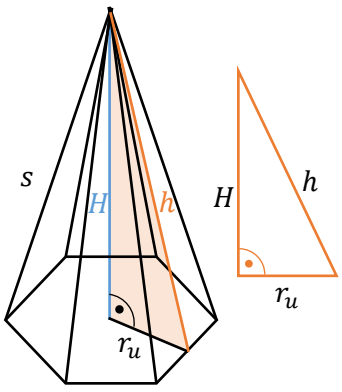
$$P_{dp} = \frac{dH}{2}$$

$$d = 2a$$



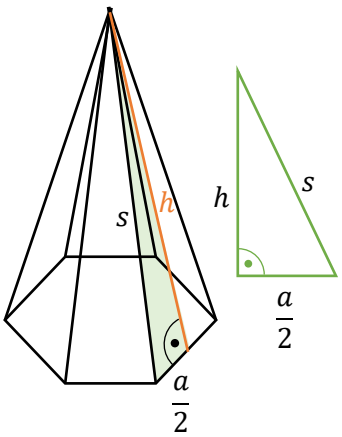
$$s^2 = H^2 + r_o^2$$

$$r_o = a$$



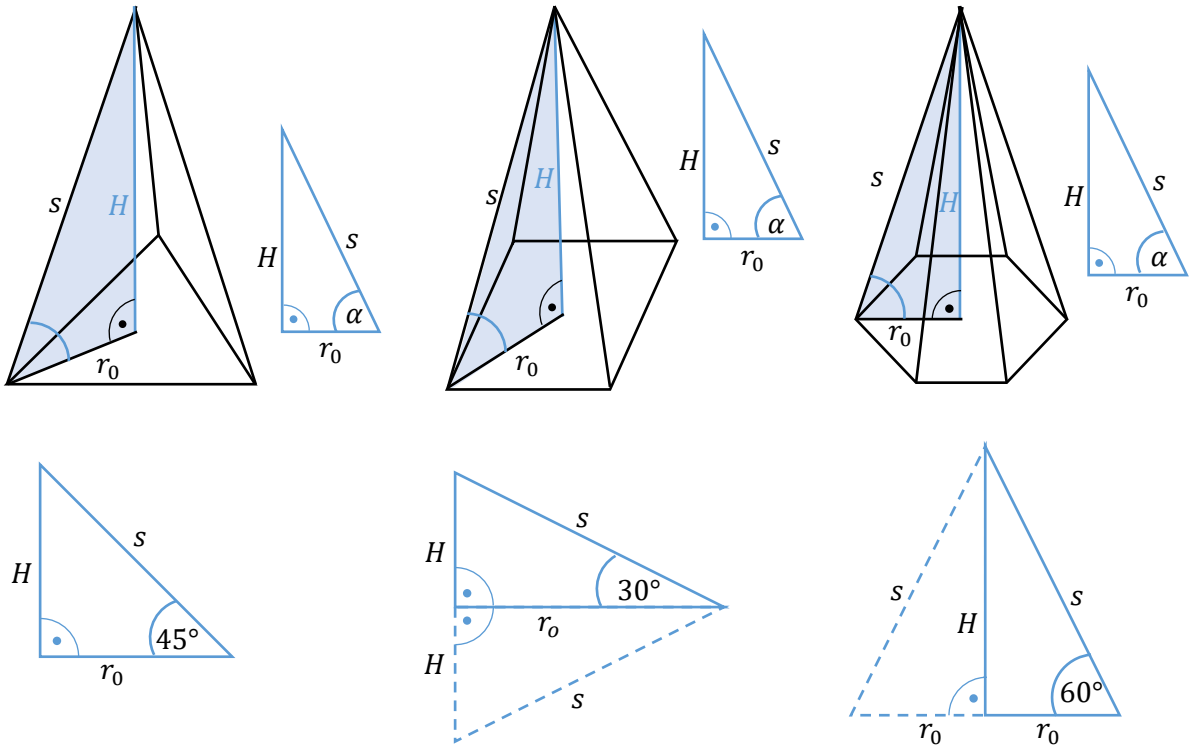
$$h^2 = H^2 + r_u^2$$

$$r_u = \frac{a\sqrt{3}}{2}$$



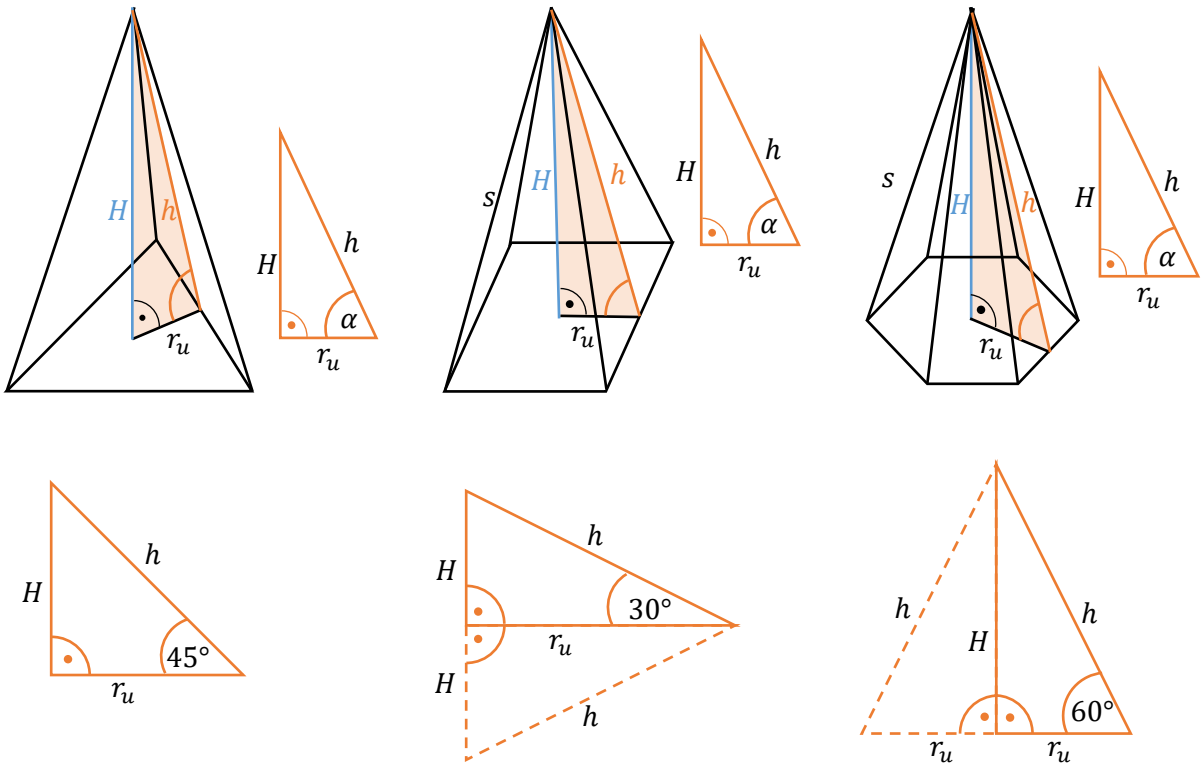
$$s^2 = h^2 + \left(\frac{a}{2}\right)^2$$

Угао између бочне ивице и равни основе $\sphericalangle(s, r_o) = \alpha, \alpha \in \{30°, 45°, 60°\}$.



$H = r_o$ $s = 2H$ $s = 2r_o$

Угао између бочне стране и равни основе $\sphericalangle(h, r_u) = \alpha, \alpha \in \{30°, 45°, 60°\}$.



$H = r_u$ $h = 2H$ $h = 2r_u$

Пример 1.

Израчунај површину и запремину праве правилне тростране пирамиде чија је основна ивица $6\sqrt{3}$ и апотема 5.

Решење:

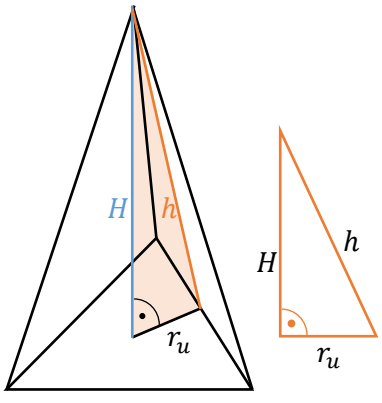
$a = 6\sqrt{3}\ cm$
$h = 5\ cm$

$$B = \frac{a^2\sqrt{3}}{4}$$
$$B = \frac{\left(6\sqrt{3}\right)^2\sqrt{3}}{4}$$
$$B = \frac{36 \cdot 3 \cdot \sqrt{3}}{4}$$
$$B = 27\sqrt{3}\ cm^2$$

$$M = 3 \cdot \frac{ah}{2}$$
$$M = 3 \cdot \frac{6\sqrt{3} \cdot 5}{2}$$
$$M = 30\sqrt{3}\ cm^2$$

$$P = B + M$$
$$P = 27\sqrt{3} + 30\sqrt{3}$$
$$P = 57\sqrt{3}\ cm^2$$

$$r_u = \frac{a\sqrt{3}}{6}$$
$$r_u = \frac{6\sqrt{3} \cdot \sqrt{3}}{6}$$
$$r_u = 3\ cm$$



$$h^2 = H^2 + r_u^2$$
$$5^2 = H^2 + 3^2$$
$$25 = H^2 + 9$$
$$H^2 = 25 - 9$$
$$H^2 = 16$$
$$H = 4\ cm$$

$$V = \frac{1}{3} \cdot BH$$
$$V = \frac{1}{3} \cdot 27\sqrt{3} \cdot 4$$
$$V = 36\sqrt{3}\ cm^3$$

Пример 2.

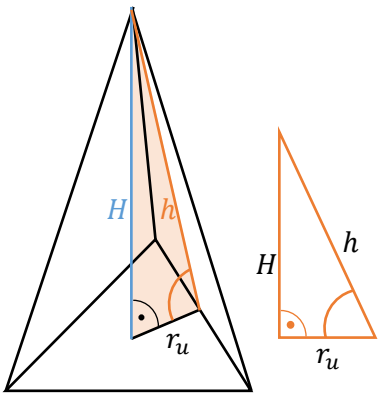
Израчунај површину праве правилне тростране пирамиде чија је основна ивица $6\ cm$ а запремина $30\sqrt{3}\ cm^3$.

Решење:

$a = 6\ cm$
$V = 30\sqrt{3}\ cm^3$

$B = \frac{a^2\sqrt{3}}{4}$
$B = \frac{6^2\sqrt{3}}{4}$
$B = \frac{36\sqrt{3}}{4}$
$B = 9\sqrt{3}$

$V = \frac{1}{3} \cdot BH$
$30\sqrt{3} = \frac{1}{3} \cdot 9\sqrt{3} \cdot H$
$30\sqrt{3} = 3\sqrt{3} \cdot H \qquad /: 3\sqrt{3}$
$H = 10\ cm$



$r_u = \frac{a\sqrt{3}}{6}$
$r_u = \frac{6\sqrt{3}}{6}$
$r_u = \sqrt{3}\ cm$

$h^2 = H^2 + r_u^2$
$h^2 = 10^2 + \left(\sqrt{3}\right)^2$
$h^2 = 100 + 3$
$h^2 = 103$
$h = \sqrt{103}\ cm$

$M = 3 \cdot \frac{ah}{2}$
$M = 3 \cdot \frac{6 \cdot \sqrt{103}}{2}$
$M = 9\sqrt{103}\ cm^2$

$P = B + M$
$P = 9\sqrt{3} + 9\sqrt{103}$
$P = 9 \cdot \left(\sqrt{3} + \sqrt{103}\right)\ cm^2$

Пример 3.

Израчунај запремину праве правилне тростране пирамиде чија је основна ивица $18\ cm$ а површина $216\sqrt{3}\ cm^2$.

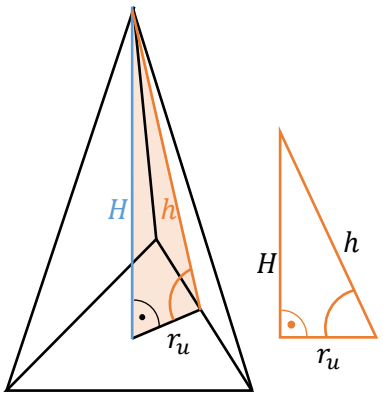
Решење:

$a = 18\ cm$
$P = 216\sqrt{3}\ cm^2$

$$B = \frac{a^2\sqrt{3}}{4}$$
$$B = \frac{18^2\sqrt{3}}{4}$$
$$B = \frac{324\sqrt{3}}{4}$$
$$B = 81\sqrt{3}\ cm^2$$

$P = B + M$
$216\sqrt{3} = 81\sqrt{3} + M$
$M = 216\sqrt{3} - 81\sqrt{3}$
$M = 135\sqrt{3}\ cm^2$

$$M = 3 \cdot \frac{ah}{2}$$
$$135\sqrt{3} = 3 \cdot \frac{18 \cdot h}{2}$$
$135\sqrt{3} = 27h \qquad /:27$
$h = 5\sqrt{3}\ cm$



$$r_u = \frac{a\sqrt{3}}{6}$$
$$r_u = \frac{18\sqrt{3}}{6}$$
$r_u = 3\sqrt{3}\ cm$

$h^2 = H^2 + r_u^2$
$\left(5\sqrt{3}\right)^2 = H^2 + \left(3\sqrt{3}\right)^2$
$25 \cdot 3 = H^2 + 9 \cdot 3$
$75 = H^2 + 27$
$H^2 = 75 - 27$
$H^2 = 48$
$H = \sqrt{48}$
$H = \sqrt{16 \cdot 3}$
$H = 4\sqrt{3}\ cm$

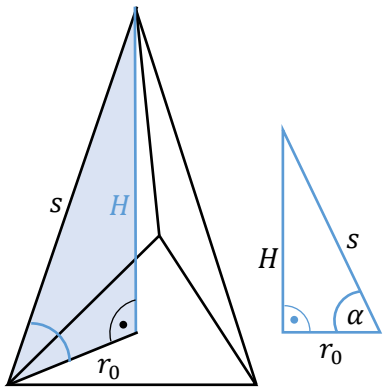
$$V = \frac{1}{3} \cdot BH$$
$$V = \frac{1}{3} \cdot 81\sqrt{3} \cdot 4\sqrt{3}$$
$$V = \frac{1}{3} \cdot 81 \cdot 3 \cdot 4$$
$V = 243\ cm^3$

Пример 4.

Израчунај запремину праве правилне тростране пирамиде чија је висина $6\,cm$ а бочна ивица са равни основе заклапа угао од 30°.

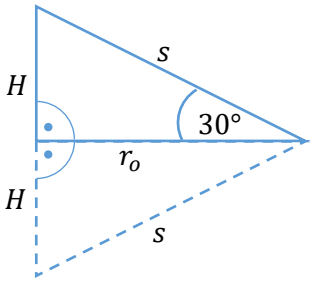
Решење:

$H = 6\,cm$
$\alpha = \sphericalangle(s, r_o) = 30°$



$s = 2H$
$s = 2 \cdot 6$
$s = 12\,cm$

$s^2 = H^2 + r_0^2$
$12^2 = 6^2 + r_o^2$
$144 = 36 + r_o^2$
$r_o^2 = 144 - 36$
$r_o^2 = 108$
$r_o = \sqrt{108}$
$r_o = \sqrt{36 \cdot 3}$
$r_o = 6\sqrt{3}\,cm$



$r_o = \frac{a\sqrt{3}}{3}$
$6\sqrt{3} = \frac{a\sqrt{3}}{3} \qquad /\cdot 3$
$18\sqrt{3} = a\sqrt{3} \qquad /:\sqrt{3}$
$a = 18\,cm$

$B = \frac{a^2\sqrt{3}}{4}$
$B = \frac{18^2\sqrt{3}}{4}$
$B = \frac{324\sqrt{3}}{3}$
$B = 108\sqrt{3}\,cm^2$

$V = \frac{1}{3} \cdot BH$
$V = \frac{1}{3} \cdot 108\sqrt{3} \cdot 6$
$V = 216\sqrt{3}\,cm^3$

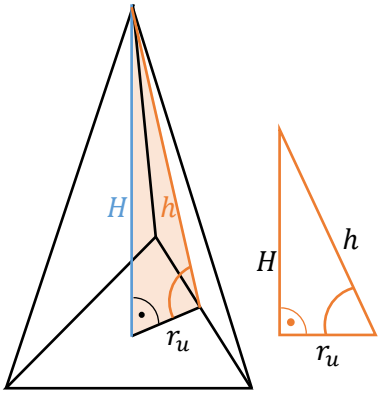
Пример 5.

Израчунај површину и запремину праве правилне тростране пирамиде чија је основна ивица 6 cm а бочна страна са равни основе заклапа угао од 60°.

Решење:

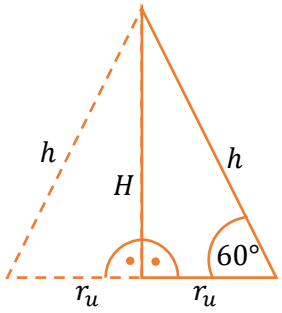
$a = 6\ cm$
$\alpha = \sphericalangle(h, r_u) = 60°$

$r_u = \frac{a\sqrt{3}}{6}$
$r_u = \frac{6\sqrt{3}}{6}$
$r_u = \sqrt{3}\ cm$



$h = 2r_u$
$h = 2 \cdot \sqrt{3}$
$h = 2\sqrt{3}\ cm$

$h^2 = H^2 + r_u^2$
$\left(2\sqrt{3}\right)^2 = H^2 + \left(\sqrt{3}\right)^2$
$4 \cdot 3 = H^2 + 3$
$12 = H^2 + 3$
$H^2 = 12 - 3$
$H^2 = 9$
$H = 3\ cm$



$B = \frac{a^2\sqrt{3}}{4}$
$B = \frac{6^2\sqrt{3}}{4}$
$B = \frac{36\sqrt{3}}{4}$
$B = 9\sqrt{3}\ cm^2$

$M = 3 \cdot \frac{ah}{2}$
$M = 3 \cdot \frac{6 \cdot 2\sqrt{3}}{2}$
$M = 18\sqrt{3}\ cm^2$

$P = B + M$
$P = 9\sqrt{3} + 18\sqrt{3}$
$P = 27\sqrt{3}\ cm^2$

$V = \frac{1}{3} \cdot BH$
$V = \frac{1}{3} \cdot 9\sqrt{3} \cdot 3$
$V = 9\sqrt{3}\ cm^3$

Пример 6.

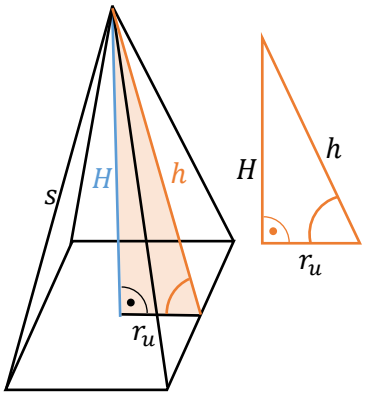
Израчунај површину и запремину праве правилне четворостране пирамиде чија је основна ивица 10 и висина 12.

Решење:

$a = 10\ cm$
$H = 12\ cm$

$B = a^2$
$B = 10^2$
$B = 100\ cm^2$

$V = \frac{1}{3} \cdot BH$
$V = \frac{1}{3} \cdot 100 \cdot 12$
$V = 400\ cm^3$



$r_u = \frac{a}{2}$
$r_u = \frac{10}{2}$
$r_u = 5\ cm$

$h^2 = H^2 + r_u^2$
$h^2 = 12^2 + 5^2$
$h^2 = 144 + 25$
$h^2 = 169$
$h = \sqrt{169}$
$h = 13\ cm$

$M = 4 \cdot \frac{ah}{2}$
$M = 4 \cdot \frac{10 \cdot 13}{2}$
$M = 260\ cm^2$

$P = B + M$
$P = 100 + 260$
$P = 360\ cm^2$

Пример 7.

Израчунај површину и запремину праве правилне четворостране пирамиде чија јњ површина основе 256 cm^2 а површина омотача 544 cm^2.

Решење:

$B = 256\ cm^2$
$M = 544\ cm^2$

$P = B + M$
$P = 256 + 544$
$P = 800\ cm^2$

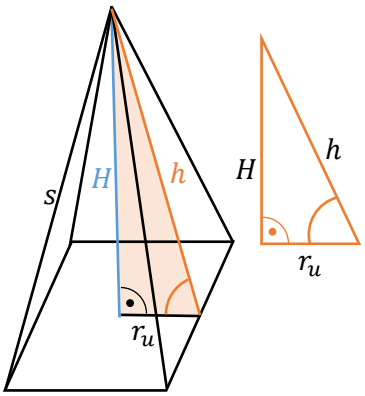
$B = a^2$
$256 = a^2$
$a = \sqrt{256}$
$a = 16\ cm$

$M = 4 \cdot \frac{ah}{2}$
$544 = 4 \cdot \frac{16 \cdot h}{2}$
$544 = 32 \cdot h \qquad /:32$
$h = 17\ cm$

$r_u = \frac{a}{2}$
$r_u = \frac{16}{2}$
$r_u = 8\ cm$

$h^2 = H^2 + r_u^2$
$17^2 = H^2 + 8^2$
$289 = H^2 + 64$
$H^2 = 289 - 64$
$H^2 = 225$
$H = \sqrt{225}$
$H = 15\ cm$

$V = \frac{1}{3} \cdot BH$
$V = \frac{1}{3} \cdot 256 \cdot 15$
$V = 1280\ cm^3$



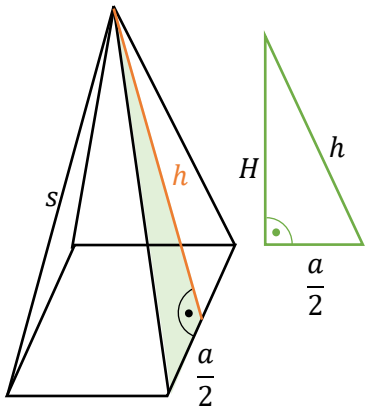
Пример 8.

Израчунај површину и запремину праве правилне четворостране пирамиде чија је бочна ивица 5 cm а површина основе 36 cm^2.

Решење:

$s = 5\ cm$
$B = 36\ cm^2$

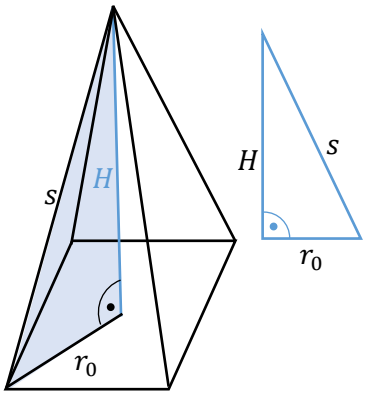
$B = a^2$
$36 = a^2$
$a = \sqrt{36}$
$a = 6\ cm$



$s^2 = h^2 + \left(\frac{a}{2}\right)^2$
$5^2 = h^2 + \left(\frac{6}{2}\right)^2$
$25 = h^2 + 9$
$h^2 = 25 - 9$
$h^2 = 16$
$h = \sqrt{16}$
$h = 4\ cm$

$d = a\sqrt{2}$
$d = 6\sqrt{2}\ cm$

$r_o = \frac{d}{2}$
$r_o = \frac{6\sqrt{2}}{2}$
$r_o = 3\sqrt{2}\ cm$



$s^2 = H^2 + r_o^2$

$5^2 = H^2 + \left(3\sqrt{2}\right)^2$
$25 = H^2 + 9 \cdot 2$
$25 = H^2 + 18$
$H^2 = 25 - 18$
$H^2 = 8$
$H = \sqrt{8}$
$H = \sqrt{4 \cdot 2}$
$H = 2\sqrt{2}\ cm$

$M = 4 \cdot \frac{ah}{2}$
$M = 4 \cdot \frac{6 \cdot 4}{2}$
$M = 48\ cm^2$

$P = B + M$
$P = 36 + 48$
$P = 84\ cm^2$

$V = \frac{1}{3} \cdot BH$
$V = \frac{1}{3} \cdot 36 \cdot 2\sqrt{2}$
$V = 24\sqrt{2}\ cm^3$

Пример 9.

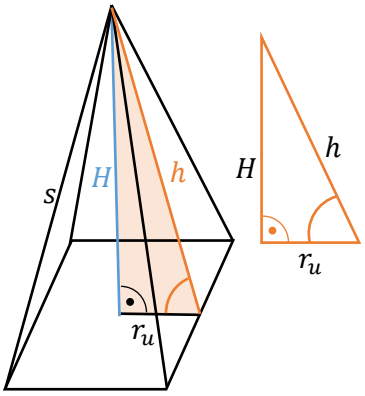
Израчунај површину и запремину праве правилне четворостране пирамиде чија је апотема $6\sqrt{2}\ cm$ и угао који бочна страна заклапа са равни основе 45°.

Решење:

$h = 6\sqrt{2}\ cm$
$\alpha = \sphericalangle(h, r_u) = 45°$

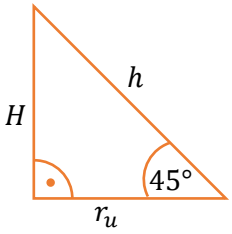
$H = r_u$

$h^2 = H^2 + r_u^2$
$\left(6\sqrt{2}\right)^2 = H^2 + H^2$
$36 \cdot 2 = 2H^2 \qquad /:2$
$H^2 = 36$
$H = \sqrt{36}$
$H = 6\ cm$



$r_u = 6\ cm$

$r_u = \frac{a}{2}$
$6 = \frac{a}{2} \qquad /\cdot 2$
$a = 12\ cm$



$B = a^2$
$B = 12^2$
$B = 144\ cm^2$

$M = 4 \cdot \frac{ah}{2}$
$M = 4 \cdot \frac{12 \cdot 6\sqrt{2}}{2}$
$M = 144\sqrt{2}\ cm^2$

$P = B + M$
$P = 144 + 144\sqrt{2}$
$P = 144 \cdot \left(1 + \sqrt{2}\right) cm^2$

$V = \frac{1}{3} \cdot BH$
$V = \frac{1}{3} \cdot 144 \cdot 6$
$V = 288\ cm^3$

Пример 10.

Израчунај запремину праве правилне четворостране пирамиде чија је бочна ивица дужине 10 cm и са равни основе заклапа угао 60°.

Решење:

$s = 10\ cm$
$\alpha = \sphericalangle(s, r_o) = 60°$

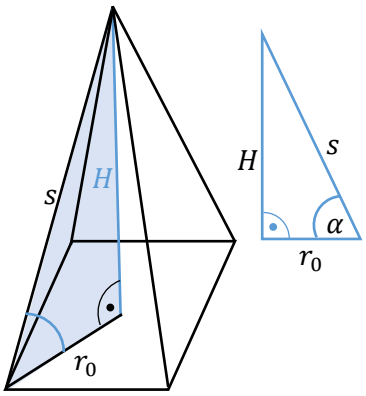
$s = 2r_o$
$10 = 2r_o \quad /:2$
$r_o = 5\ cm$

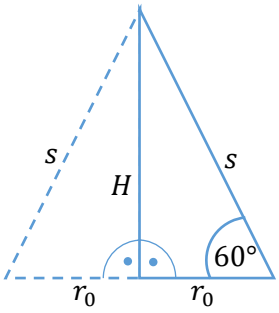
$r_o = \frac{d}{2}$
$5 = \frac{d}{2} \quad /\cdot 2$
$d = 10\ cm$

$d = a\sqrt{2}$
$10 = a\sqrt{2}$
$a = \frac{10}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$
$a = \frac{10\sqrt{2}}{2}$
$a = 5\sqrt{2}\ cm$

$s^2 = H^2 + r_o^2$
$10^2 = H^2 + 5^2$
$100 = H^2 + 25$
$H^2 = 100 - 25$
$H^2 = 75$
$H = \sqrt{75}$
$H = \sqrt{25 \cdot 3}$
$H = 5\sqrt{3}\ cm$

$B = a^2$
$B = \left(5\sqrt{2}\right)^2$
$B = 25 \cdot 2$
$B = 50\ cm^2$





$V = \frac{1}{3} \cdot BH$
$V = \frac{1}{3} \cdot 50 \cdot 5\sqrt{3}$
$V = \frac{250\sqrt{3}}{3}\ cm^3$

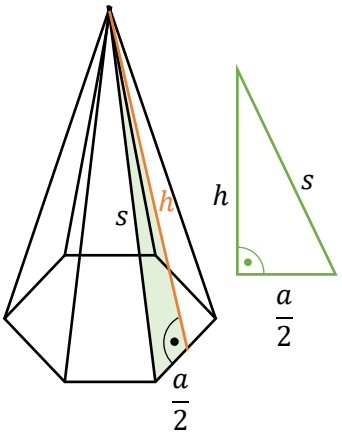
Пример 11.

Израчунај запремину праве правилне шестостране пирамиде ако је бочна ивица 13 cm и висина бочне стране 12 cm.

Решење:

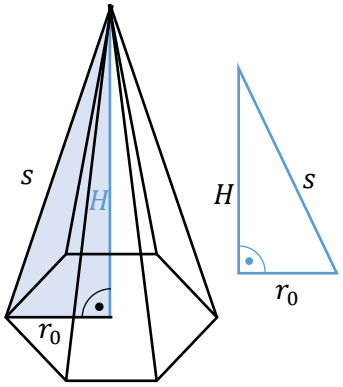
$s = 13\ cm$
$h = 12\ cm$

$$s^2 = h^2 + \left(\frac{a}{2}\right)^2$$
$$13^2 = 12^2 + \left(\frac{a}{2}\right)^2$$
$$169 = 144 + \left(\frac{a}{2}\right)^2$$
$$\left(\frac{a}{2}\right)^2 = 169 - 144$$
$$\left(\frac{a}{2}\right)^2 = 25$$
$$\frac{a}{2} = \sqrt{25}$$
$$\frac{a}{2} = 5$$
$$a = 10\ cm$$



$$r_o = a$$
$$r_o = 10\ cm$$

$$s^2 = H^2 + r_o^2$$
$$13^2 = H^2 + 10^2$$
$$169 = H^2 + 100$$
$$H^2 = 169 - 100$$
$$H^2 = 69$$
$$H = \sqrt{69}\ cm$$



$$B = 6 \cdot \frac{a^2\sqrt{3}}{4}$$
$$B = 6 \cdot \frac{10^2\sqrt{3}}{4}$$
$$B = 6 \cdot \frac{100\sqrt{3}}{4}$$
$$B = 150\sqrt{3}\ cm^2$$

$$V = \frac{1}{3} \cdot BH$$
$$V = \frac{1}{3} \cdot 150\sqrt{3} \cdot \sqrt{69}$$
$$V = \frac{1}{3} \cdot 150\sqrt{3} \cdot \sqrt{23 \cdot 3}$$
$$V = \frac{1}{3} \cdot 150 \cdot \sqrt{3} \cdot \sqrt{23} \cdot \sqrt{3}$$
$$V = \frac{1}{3} \cdot 150 \cdot 3 \cdot \sqrt{23}$$
$$V = 150\sqrt{23}\ cm^3$$

Пример 12.

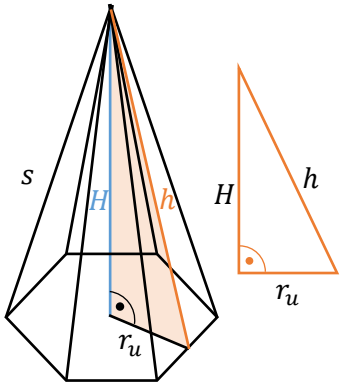
Израчунај површину и запремину праве правилне шестостране пирамиде чија је основна ивица 8 cm а површина омотача $120\sqrt{3}\ cm^2$.

Решење:

$a = 8\ cm$
$M = 120\sqrt{3}\ cm^2$

$M = 6 \cdot \frac{ah}{2}$
$120\sqrt{3} = 6 \cdot \frac{8 \cdot h}{2}$
$120\sqrt{3} = 24 \cdot h \qquad /:24$
$h = 5\sqrt{3}\ cm$

$r_u = \frac{a\sqrt{3}}{2}$
$r_u = \frac{8\sqrt{3}}{2}$
$r_o = 4\sqrt{3}\ cm$



$h^2 = H^2 + r_u^2$
$\left(5\sqrt{3}\right)^2 = H^2 + \left(4\sqrt{3}\right)^2$
$25 \cdot 3 = H^2 + 16 \cdot 3$
$75 = H^2 + 48$
$H^2 = 75 - 48$
$H^2 = 28$
$H = \sqrt{28}$
$H = \sqrt{4 \cdot 7}$
$H = 2\sqrt{7}\ cm$

$B = 6 \cdot \frac{a^2\sqrt{3}}{4}$
$B = 6 \cdot \frac{8^2\sqrt{3}}{4}$
$B = 6 \cdot \frac{64\sqrt{3}}{4}$
$B = 96\sqrt{3}\ cm^2$

$P = B + M$
$P = 96\sqrt{3} + 120\sqrt{3}$
$P = 216\sqrt{3}\ cm^2$

$V = \frac{1}{3} \cdot BH$
$V = \frac{1}{3} \cdot 96\sqrt{3} \cdot 2\sqrt{7}$
$V = 64\sqrt{21}\ cm^3$

Пример 13.

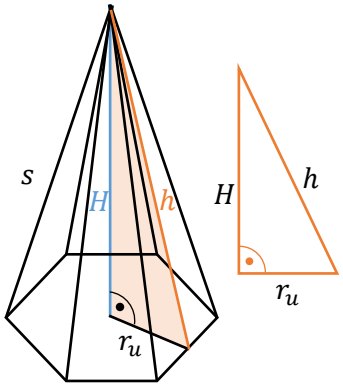
Израчунај површину праве правилне шестостране пирамиде чија је висина $3\sqrt{3}\ cm$ а запремина $162\ cm^3$.

Решење:

$H = 3\sqrt{3}\ cm$
$V = 162\ cm^3$

$V = \frac{1}{3} \cdot BH$
$162 = \frac{1}{3} \cdot B \cdot 3\sqrt{3}$
$162 = B \cdot \sqrt{3}$
$B = \frac{162}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$
$B = \frac{162\sqrt{3}}{3}$
$B = 54\sqrt{3}\ cm^2$

$B = 6 \cdot \frac{a^2\sqrt{3}}{4}$
$54\sqrt{3} = \frac{6a^2\sqrt{3}}{4} \qquad / \cdot 4$
$6a^2\sqrt{3} = 216\sqrt{3} \qquad / : 6\sqrt{3}$
$a^2 = 36$
$a = \sqrt{36}$
$a = 6\ cm$



$r_u = \frac{a\sqrt{3}}{2}$
$r_u = \frac{6\sqrt{3}}{2}$
$r_u = 3\sqrt{3}\ cm$

$h^2 = H^2 + r_u^2$
$h^2 = \left(3\sqrt{3}\right)^2 + \left(3\sqrt{3}\right)^2$
$h^2 = 9 \cdot 3 + 9 \cdot 3$
$h^2 = 27 + 27$
$h^2 = 54$
$h = \sqrt{54}$
$h = \sqrt{9 \cdot 6}$
$h = 3\sqrt{6}\ cm$

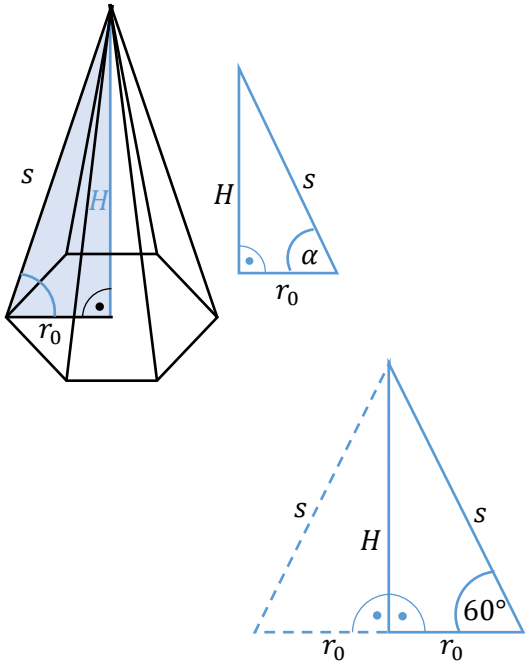
$M = 6 \cdot \frac{ah}{2}$
$M = 6 \cdot \frac{6 \cdot 3\sqrt{6}}{2}$
$M = 54\sqrt{6}\ cm^2$

$P = B + M$
$P = 54\sqrt{3} + 54\sqrt{6}$
$P = 54\sqrt{3} \cdot \left(1 + \sqrt{2}\right)\ cm^2$

Пример 14.

Израчунај површину и запремину праве правилне шестостране пирамиде чија је основна ивица 4 cm а угао између бочне ивице и равни основе 60°.

Решење:

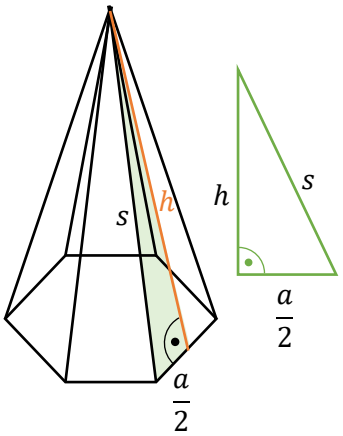


$a = 4\ cm$
$\alpha = \sphericalangle(s, r_o) = 60°$

$r_o = a$
$r_o = 4\ cm$

$s = 2r_o$
$s = 2 \cdot 4$
$s = 8\ cm$

$s^2 = H^2 + r_o^2$
$8^2 = H^2 + 4^2$
$64 = H^2 + 16$
$H^2 = 48$
$H = \sqrt{48}$
$H = \sqrt{16 \cdot 3}$
$H = 4\sqrt{3}$



$s^2 = h + \left(\frac{a}{2}\right)^2$
$8^2 = h^2 + \left(\frac{4}{2}\right)^2$
$64 = h^2 + 4$
$h^2 = 64 - 4$
$h^2 = 60$
$h = \sqrt{60}$
$h = \sqrt{4 \cdot 15}$
$h = 2\sqrt{15}\ cm$

$B = 6 \cdot \frac{a^2\sqrt{3}}{4}$
$B = 6 \cdot \frac{4^2\sqrt{3}}{4}$
$B = 6 \cdot \frac{16\sqrt{3}}{4}$
$B = 24\sqrt{3}\ cm^2$

$M = 6 \cdot \frac{ah}{2}$
$M = 6 \cdot \frac{4 \cdot 2\sqrt{15}}{2}$
$M = 24\sqrt{15}\ cm^2$

$P = B + M$
$P = 24\sqrt{3} + 24\sqrt{15}$
$P = 14\sqrt{3} \cdot \left(1 + \sqrt{5}\right)\ cm^2$

$V = \frac{1}{3} \cdot BH$
$V = \frac{1}{3} \cdot 24\sqrt{3} \cdot 4\sqrt{3}$
$V = \frac{1}{3} \cdot 24 \cdot 3 \cdot 4$
$V = 96\ cm^3$

Пример 15.

Израчунај површину и запремину праве правилне шестостране пирамиде чија је основна ивица 6 cm а угао између бочне стране и равни основе 30°.

Решење:

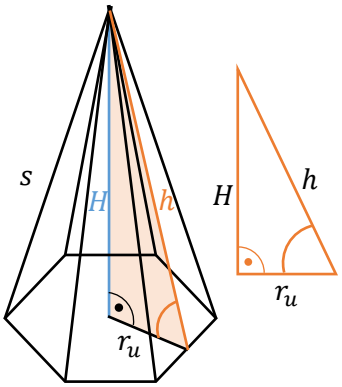
$a = 6\ cm$
$\alpha = \sphericalangle(h, r_u) = 30°$

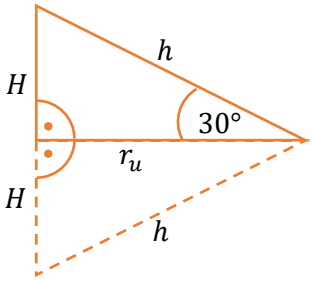
$h = 2H$

$r_u = \frac{a\sqrt{3}}{2}$
$r_u = \frac{6\sqrt{3}}{2}$
$r_u = 3\sqrt{3}\ cm$

$h^2 = H^2 + r_u^2$
$(2H)^2 = H^2 + r_u^2$
$4H^2 = H^2 + r_u^2$
$4H^2 - H^2 = \left(3\sqrt{3}\right)^2$
$3H^2 = 9 \cdot 3$
$H^2 = 9$
$H = 3\ cm$

$h = 2 \cdot 3$
$h = 6\ cm$





$B = 6 \cdot \frac{a^2\sqrt{3}}{4}$
$B = 6 \cdot \frac{6^2\sqrt{3}}{4}$
$B = 6 \cdot \frac{36\sqrt{3}}{4}$
$B = 54\sqrt{3}\ cm^2$

$M = 6 \cdot \frac{ah}{2}$
$M = 6 \cdot \frac{6 \cdot 6}{2}$
$M = 108\ cm^2$

$P = B + M$
$P = 54\sqrt{3} + 108$
$P = 54 \cdot \left(\sqrt{3} + 2\right)\ cm^2$

$V = \frac{1}{3} \cdot BH$
$V = \frac{1}{3} \cdot 54\sqrt{3} \cdot 3$
$V = 54\sqrt{3}\ cm^3$